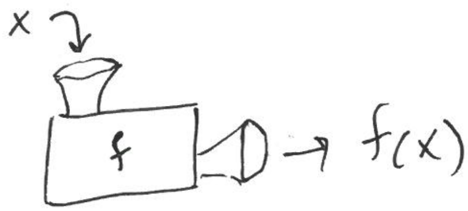
What is a function?



takes an input over some domain and produces a unique output.

ex/ $f(x) = x + e^x$

input any real number x, output $x + e^x$
(Domain is $\mathbb{R}$)

range? $\mathbb{R}$

ex/ $f(x) = \frac{1}{x}$

what happens if we plug in 0?

Domain: $x \neq 0$
or
$\{x \in \mathbb{R} \mid x \neq 0\}$

Range: any real number not equal to 0

ex/ $f(x) = x^2$

Domain: $x \in \mathbb{R}$

Range: $f(x) \geq 0$

ex/ $g(x) = \sqrt{x+2}$

Domain:

$x \geq -2$

Range: $g(x) \geq 0$

Ex/

x	$f(x)$
0	8
1	9
2	7
3	6
4	6

Domain

$\{0, 1, 2, 3, 4\}$

Range

$\{8, 9, 7, 6\}$

ex/

x	$f(x)$
0	1
2	2
2	3
3	1
4	2

Not a function!

$f(2) = 2$
and
$f(2) = 3$

In Summary Domain is what we are allowed to plug in, Range is what is possible to get as output.

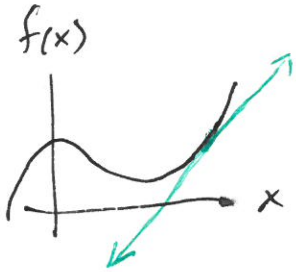
The derivative:

$$f(x) = y$$

$$f'(x),\ y',\ \frac{df}{dx},\ \ldots = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

"The rate of change of $f(x)$ at x"

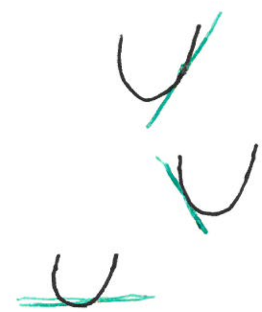
"The slope of the tangent line to the graph of f"



If $f'(x) > 0$, f increasing at x

$f'(x) < 0$, f decreasing at x

$f'(x) = 0$, critical point



Maxima and Minima occur at critical points

Computing the derivative:

Power rule: $f(x) = x^a$

$f'(x) = a x^{a-1}$

ex/

- $f(x) = x^2$ $\qquad f'(x) = 2x$
- $f(x) = \frac{1}{x}$

  $= x^{-1}$ $\qquad f'(x) = -x^{-2}$

  $= -\frac{1}{x^2}$
- $f(x) = \sqrt[3]{x}$ $\qquad f'(x) = \frac{1}{3} x^{-2/3}$

  $= x^{1/3}$

  $= \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$
- $f(x) = \frac{1}{-\sqrt{x}}$
- $f(x) = x^2 + 3x^3$
- $f(x) = 2\sqrt{x} + 3\sqrt[3]{x}$
- $f(x) = 3$ $\qquad f'(x) = 0$

Some special functions.

- $f(x) = e^x$ $\quad f'(x) = e^x$
- $f(x) = \ln(x)$ $\quad f'(x) = \frac{1}{x}$
- $f(x) = \sin(x)$ $\quad f'(x) = \cos(x)$
- $f(x) = \cos(x)$ $\quad f'(x) = -\sin(x)$

Product Rule:

$$(fg)' = f'g + fg'$$

- $f(x) = e^x x^5$ $\quad f'(x) = e^x x^5 + 5e^x x^4$
- $f(x) = x^2 x^4$ $\quad f'(x) = 2x \cdot x^4 + 4x^2 x^3$

$= x^6$ $\quad = 2x^5 + 4x^5 = 6x^5$

Chain Rule:

$$(f(g(x)))' = f'(g(x))\, g'(x)$$

"Derivative of outside times derivative of inside"

- $f(x) = \sqrt{x^2+3}$ $\quad f'(x) = \frac{1}{2\sqrt{x^2+3}} \cdot 2x = \frac{x}{\sqrt{x^2+3}}$
- $f(x) = e^{3x}$ $\quad f'(x) = e^{3x}(3)$
- $f(x) = e^{2x^3+2x^2+5}$ $\quad f'(x) = e^{2x^3+2x^2+5}(6x^2+4x)$

Quotient Rule:

$$\left(\frac{f(x)}{g(x)}\right)' = \cancel{g f(x)} \; \frac{g f' - f g'}{g^2}$$

~~" low d(high) - high d~~

" $\dfrac{\text{low } d(\text{high}) - \text{high } d(\text{low})}{\text{low low}}$ "

- $f(x) = \dfrac{x^4}{2x + e^{3x}}$ $\qquad f'(x) = \dfrac{(2x + e^{3x})(4x^3) - (x^4)(2+3e}{(2x + e^{3x})^2}$

- $f(x) = \dfrac{e^x}{x}$ $\qquad f'(x) = \dfrac{x e^x - e^x}{x^2}$

2nd Derivative:

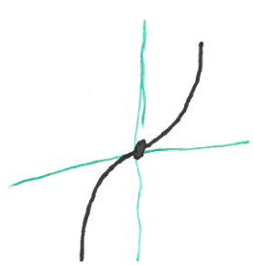
"acceleration" "rate-of-change of rate-of-change"

$f''(x) > 0$ graph is concave up

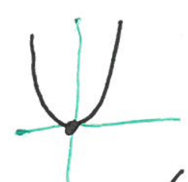
$f''(x) < 0$ graph is concave down

$f''(x) = 0$ inflection point

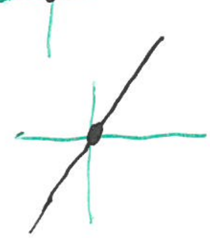
ex $f(x) = x^3$



$f'(x) = 3x^2$



$f''(x) = 6x$



$$\cancel{h(x) = \frac{\ln\left(\sqrt{e^{3x}}\right)\left(5x^2+9\right)}{\sqrt[3]{x^3+x^2}}}$$

$$f(x) = \left(\frac{x^5 - 5x^3 + 2x}{x^3}\right)$$

$$\cancel{(x^5 - 5x^3 + 2x)}$$

$$\boxed{f'(x) = \frac{x^3(5x^4 - 15x^2 + 2) - (x^5 - 5x^3 + 2x)(3x^2)}{x^6}}$$

$$f(x) = \frac{x^5}{x^3} - \frac{5x^3}{x^3} + \frac{2x}{x^3} = x^2 - 5 + \frac{2}{x^2}$$

$$\boxed{f'(x) = 2x - \frac{4}{x^3}}$$